

This week, we pick from where we left [last week](#). Let's discuss the last 3 cases first.

Question: Which set, S or T, has higher SD?

Case 5:  $S = \{1, 3, 5\}$  or  $T = \{1, 3, 3, 5\}$

The standard deviation (SD) of T will be less than the SD of S. Why? The mean of 1, 3 and 5 is 3. If you add another 3 to the list, the mean stays the same and the sum of the squared deviations is also the same but the number of elements increases. Hence, the SD decreases.

Case 6:  $S = \{6, 8, 10\}$  or  $T = \{12, 16, 20\}$

Put the numbers on the number line. You will see that the SD of T is greater than the SD of S. When you multiply each element of a set by the same number (T is obtained by multiplying each element of S by 2), the SD increases.

Case 7:  $S = \{6, 8, 10\}$  or  $T = \{3, 4, 5\}$

Put the numbers on the number line. You will see that the SD of T is less than the SD of S. When you divide each element of a set by the same number (T is obtained by dividing each element of S by 2 OR you can say that S is obtained by multiplying each element of T by 2), the SD decreases.

Now that we have an understanding of how SD behaves, let's look at a question.

**Question 1:** A certain list of 300 test scores has an arithmetic mean of 75 and a standard deviation of  $d$ , where  $d$  is positive. Which of the following two test scores, when added to the list, must result in a list of 302 test scores with a standard deviation less than  $d$ ?

- (A) 75 and 80
- (B) 80 and 85
- (C) 70 and 75
- (D) 75 and 75
- (E) 70 and 80

**Solution:** As discussed last week, the standard deviation of a set measures the deviation from the mean. A low standard deviation indicates that the data points are very close to the mean whereas a high standard deviation indicates that the data points are spread far apart from the mean.

When we add numbers that are far from the mean, we are stretching the set and hence, increasing the SD. When we add numbers which are close to the mean, we are shrinking the set and hence, decreasing the SD.

Therefore, **adding two numbers which are closest to the mean will shrink the set the most**, thus decreasing SD by the greatest amount.

Numbers closest to the mean are 75 and 75 (they are equal to the mean) and thus adding them will decrease SD the most.

Answer: D.

Now that we have seen that difficult looking questions on SD can be quite simple, I want you to think about something – when you add some new numbers to a set, how do you decide whether SD increases or decreases? If you notice, we have seen two different cases (case 4 and case 5) – in one of them SD increases when you add two numbers to the set

and in the other, SD decreases. So how do you decide whether SD will increase or decrease? Say, what happens in case  $S = \{3, 4, 5, 6, 7\}$  and  $T = \{3, 4, 4, 5, 6, 6, 7\}$ ? Will SD increase or decrease in this case? How do you decide the point at which the increase in the numerator offsets the increase in the denominator?

Meanwhile, let's look at one more question.

**Question 2:** If 100 is included in each of sets A, B and C (given  $A = \{30, 50, 70, 90, 110\}$ ,  $B = \{-20, -10, 0, 10, 20\}$  and  $C = \{30, 35, 40, 45, 50\}$ ), which of the following represents the correct ordering (largest to smallest) of the sets in terms of the absolute increase in their standard deviation?

- (A) A, C, B
- (B) A, B, C
- (C) C, A, B
- (D) B, A, C
- (E) B, C, A

**Solution:** The question looks a little convoluted but actually you don't have to calculate anything. SD measures the deviation of the elements from the mean. If a new element is added which is far away from the mean, it will add much more to the deviations than if it were added close to the mean.

The means of A, B and C are 70, 0 and 40, respectively.

100 is farthest from 0 so it will change the SD of set B the most (in terms of absolute increase). It is closest to 70 so it will change the SD of set A the least. Hence the correct ordering is B, C, A.

Answer (E)

Simple enough, right? SD questions are generally straight forward once you understand the basics well. See you next week with a tricky SD question!